Critical behavior of the two-dimensional randomly driven lattice gas

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We investigate the critical behavior of the two-dimensional randomly driven lattice gas, in which particles are driven along one of the lattice axes by an infinite external field with randomly changing sign. A finite-size scaling (FSS) analysis provides novel evidences that this model is not in the same universality class as the driven lattice gas with a constant drive (DLG), contrarily to what has been recently reported in the literature. Indeed, the FSS functions of transverse observables (i.e., related to order-parameter fluctuations with wave vector perpendicular to the direction of the field) differ from the mean-field behavior—both predicted and observed in the DLG. In sharp contrast to the case of the DLG, FSS can be established only for rectangular lattices where the dimension in the direction of the field grows as the second power of the other dimension. Further, the transverse Binder cumulant does not vanish at the critical point.

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I. INTRODUCTION

Phase transitions are characterized by a drastic change in the macroscopic behavior of many-body interacting systems when control parameters are varied. In the case of critical phenomena the onset of the ordered phase is accompanied by fluctuations on all length scales. In spite of the difficulties in accounting efficiently for such coupled fluctuations, we have currently a deep and detailed understanding of the collective behavior of a large class of systems in thermal equilibrium thanks to a variety of results and methods, both analytical and numerical. Critical collective behaviors, on the other hand, are also observed in the steady states of systems far from thermal equilibrium [1]. In contrast to equilibrium cases, the stationary probability distribution is not known *a priori* and the possible occurrence and nature of a phase transition can no longer be determined by usual entropy-energy arguments. The absence of detailed balance, the generic algebraic decay of space-dependent correlations as functions of the distance, their strongly anisotropic scaling properties, and the flux of particles or energy through the system are among the general features which make these problems particularly difficult and rich in phenomenology. Because of the lack of a general framework, it is still worth focusing on specific toy models in order to gain insight which might possibly lead to a more comprehensive theory.

Among the models characterized by nonequilibrium steady states the simplest is the uniformly driven lattice gas [2] (DLG), a generalization of the Ising model to nonequilibrium conditions due to the action of an external nonconservative force, inducing a particle current through the system. Although the DLG was introduced more than 20 years ago, there is still room for debate on the nature and properties (in particular, the universality class) of the nonequilibrium critical behavior observed upon changing the temperature. At first, the relevant feature of the model was assumed to be the presence of a particle current [3]. However, more recently, this point has been criticized by arguing that the strong anisotropy is, instead, its qualifying character [4,5]. In addition to the theoretical debate [6], seemingly contradictory evidences are also coming from Monte Carlo (MC) simulations [5,7–9].

The Ising model can be driven to nonequilibrium conditions also by the coupling to two thermal baths at different temperatures [10], controlling the hopping rates of the particles in different lattice directions. In a simpler version, the temperature affecting the jumps in one direction is taken to be infinite. Hereafter we will refer to this direction as the longitudinal one ($\parallel$) whereas we refer to the remaining as transverse ones ($\perp$). This model is equivalent to a DLG in which the microscopic external driving force is along the longitudinal direction, with infinite modulus and a sign that is randomly chosen for each lattice site every time step (annealed randomness). The resulting model is called the randomly driven lattice gas (RDLG) [11]. Unlike the DLG, no net particle current is flowing through the system. MC simulations [12] indicate that in the RDLG transverse fluctuations of the order parameter (i.e., the fluctuations with wave vector in the transverse direction) are not effectively described by a Gaussian model. Indeed, in two dimensions, the case we shall consider from now on, MC simulations give $\nu_\perp=0.62(3)$ and $\eta=0.13(4)$ [12]. These estimates rely on a field-theoretical result for the *anisotropy exponent* $\Delta\equiv(\nu_\parallel/\nu_\perp)-1$, i.e., $\Delta=1-\eta/2\approx 1$ [11] which enters the finite-size scaling (FSS) *Ansätze* used to extract critical exponents. In Ref. [5] the numerical data for the RDLG and the DLG on the same finite lattices have been compared. According to these data the two models have the same finite-size scaling (FSS) behavior. If true, this implies that they belong to the same universality class and thus the strong anisotropy and not the particle current is the relevant feature in determining the leading critical behavior of driven diffusive systems. The same conclusions has been drawn in Ref.

[7] by studying the short-time dynamics, although some points of the analysis therein remain unclear [13]. Here we reconsider the problem presenting the results of a new series of MC simulations of the RDLG. The critical behavior of the system (proper of the thermodynamic limit) is extracted from data on finite lattices by means of a FSS analysis that does not require free parameters [14], in contrast with that previously employed [15].

II. THE MODEL

We briefly recall the definition of the RDLG. Consider a rectangular lattice and for each site x introduce an occupation variable n_x, which can be either zero (empty site) or one (occupied site). The external field E is acting along the longitudinal direction but with random sign. The dynamics of the model is of Kawasaki type: A lattice link $\langle xy\rangle$ is randomly chosen, and, if $n_x \neq n_y$, a particle jump is proposed and then accepted with Metropolis probability $w(\beta\Delta H + \beta E\ell)$, where $\ell=(1,0,-1)$ for jumps (along, transverse, opposite) to E, $w(x)=\min(1,e^{-x})$, and ΔH is the variation of the standard lattice-gas nearest-neighbor attractive interaction $H=-4\Sigma_{\langle xy\rangle}n_x n_y$ due to the proposed jump. The parameter β plays the role of an inverse temperature. In the DLG, E is constant and time-independent. Periodic boundary conditions in the direction of E make it a nonconservative field and drive the system into a nonequilibrium stationary state. Although the boundary conditions are not so relevant in the RDLG, we will assume them periodic in all directions.

At half filling, the RDLG undergoes a second-order phase transition. Indeed, at high temperatures the steady state is disordered whereas at low temperatures the system orders: The particles condense forming a strip with interfaces parallel to E. These two phases are separated by a phase transition occurring at the critical value $\beta_c(E)$ depending on the field E. Here we will concentrate on the particular case in which E is infinite.

III. FINITE-SIZE SCALING ANALYSIS

For a strongly anisotropic system in d dimensions, with finite size $L_\parallel \times L_\perp^{d-1}$, the FSS limit corresponds to $t\equiv 1-\beta/\beta_c \to 0$ (where β_c is the bulk critical temperature), $L_\parallel, L_\perp \to \infty$, keeping fixed both combinations $tL_\parallel^{1/\nu_\parallel}$ and $tL_\perp^{1/\nu_\perp}$, and therefore also the so-called "aspect ratio" $S_\Delta = L_\parallel^{1/(1+\Delta)}/L_\perp$ [16]. Accordingly, the FSS analysis of numerical data generally requires an *a priori* knowledge of the exponent Δ [17]. It would be a real step towards a better understanding of nonequilibrium phase transitions to have FSS in a form suitable for these systems, reliable and powerful enough to disentangle those key features which might be buried in tiny differences when the volume of the samples is increased. In this direction we have already performed a detailed study of the FSS of the DLG [9] by using the general strategy introduced in Ref. [14], confirming the mean-field behavior of transverse fluctuations, with $\Delta=2$, in agreement with the predictions of Ref. [3]. It is therefore a crucial test to examine by the same method also the RDLG. For previous studies of FSS in strongly anisotropic systems see Refs. [15,16]. We will show that the method is sensitive enough to highlight the differences in the critical behavior of the DLG and RDLG (contrarily to the claims in Ref. [18]), to an extent that goes beyond the numerical differences in the critical exponents and probes the spatial structure of correlations.

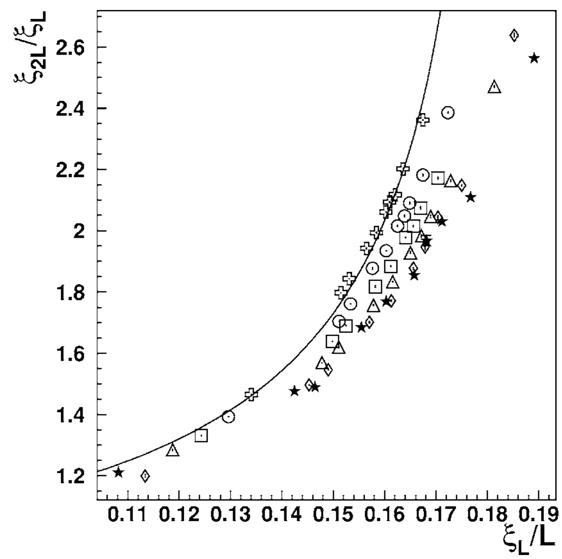

FIG. 1. FSS plot of the transverse correlation length ξ_L in the two-dimensional RDLG with fixed $S_2 \approx 0.200$. Crosses, circles, squares, triangles, diamonds, and stars correspond to lattices of increasing size $L=14,16,18,20,22,24$. The Gaussian behavior approached by the DLG is given by the solid line.

In spite of the generic power-law decay of the two-point correlation function $\langle n_x n_0\rangle$ for large x [12], it is possible to define a finite-volume correlation length [9]. Given the Fourier transform $G(q)$ of $\langle n_x n_0\rangle$, one considers $G_\perp(q)\equiv G(\{q_\parallel=0, q_\perp=q\})$ and defines the correlation length

$$\xi_L \equiv \sqrt{\frac{1}{\hat{q}_3^2-\hat{q}_1^2}\left[\frac{G_\perp(q_1)}{G_\perp(q_3)}-1\right]}, \tag{1}$$

where $\hat{q}_n = 2\sin q_n/2$ is the lattice momentum and $q_n = 2\pi n/L_\perp$. Hereafter we will denote $L_\perp$ simply by L.

In Fig. 1 we report the FSS plot of the ratio ξ_{2L}/ξ_L, where ξ_{2L} and ξ_L are computed at the same temperature but lattice sizes $2L$ and L, respectively, keeping constant the aspect ratio with $\Delta=2$, i.e., $S_2 \simeq 0.200$ [19]. For comparison we report as a solid line the mean-field prediction which is approached by the DLG data on larger lattices [9]. In the present case, deviations from the mean-field behavior increase with increasing lattice sizes. Note that, if S_1 is the correct aspect ratio for the model, then one observes the crossover towards the FSS of the model in the strip geometry $L_\perp=\infty$, when keeping S_2 constant and ξ_L/L fixed [17]. Accordingly, the points corresponding to larger lattices in Fig. 1 eventually accumulate on some limiting curve as L increases, in agreement with the predictions [20] based on the field-theoretical model of Ref. [11].

Figure 2 refers to geometries with $\Delta=1$, i.e., $S_1 \approx 0.223$ (upper set of points) and $S_1 \approx 0.326$ (lower set). Note that we

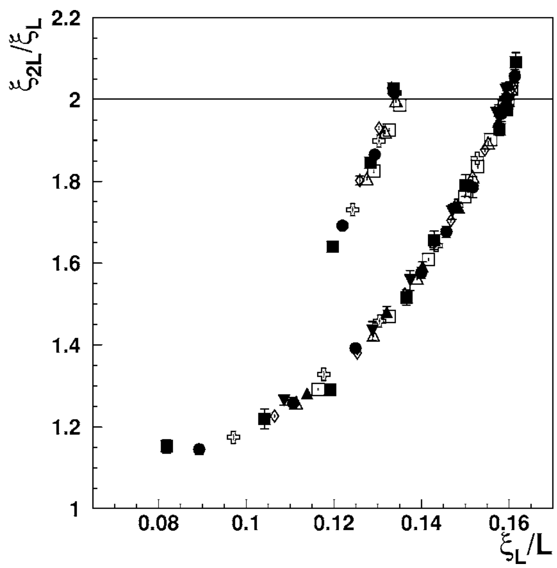

FIG. 2. FSS plot of the transverse correlation length ξ_L in the two-dimensional RDLG with fixed $S_1=0.223$ (upper set of points) and $S_1=0.326$ (lower set). Empty squares, empty triangles, diamonds, crosses, full circles, full squares, full upright triangles, and full downright triangles correspond to lattices of increasing size $L=20,22,24,28,32,36,40,44$.

used $\Delta=1$, although a correction of the order of $\eta/2$ to this value is expected. In these cases we have been able to reach $L=88$. In contrast with the mean-field behavior, which does not depend on the specific value of S_Δ, now we do expect a dependence of the FSS curves on the actual value of S_1. The critical properties can be extracted from the previous plot as follows. For a given S_1, consider the scaling function $\xi_{2L}/\xi_L=F(z)$ as a functions of $z=\xi_L/L$ and expand it around z^*, which is defined as the point such that $F(z^*)=2$, i.e., as the value of ξ_L/L at the critical temperature. Denoting $\delta z=z-z^*$, one finds [9]

$$F(z) = F(z^*) + F'(z^*)\delta z + O[(\delta z)^2]$$
$$= 2 + \frac{2}{z^*}(2^{1/\nu_\perp} - 1)\delta z + O[(\delta z)^2]. \quad (2)$$

A linear fit of our data gives $z^*=0.1337(3)$ for $S_1=0.223$ and $z^*=0.1594(1)$ for $S_1=0.326$, with the same

$$\nu_\perp = 0.61(3). \quad (3)$$

The corresponding critical temperature is the same in the two cases. Note that z^* for $S_1=0.326$ is almost equal to the mean-field value $1/(2\pi)$ [9]. Indeed, we had chosen this value of S_1 in order to be very close to the mean-field predictions and test whether the FSS method employed is able to detect the differences. These results suggest that unlike the DLG, where mean-field scaling at fixed S_2 is observed, in the present case scaling is attained only at fixed S_1 and is not compatible with mean-field behavior. Indeed, not only does z^* depend on the geometry, but also the critical exponent $\nu_\perp$ differs from the Gaussian value $1/2$. The qualitative dependence of z^* on S_1 is accounted for [20] by the field-theoretical model of Ref. [11]. A similar analysis, with

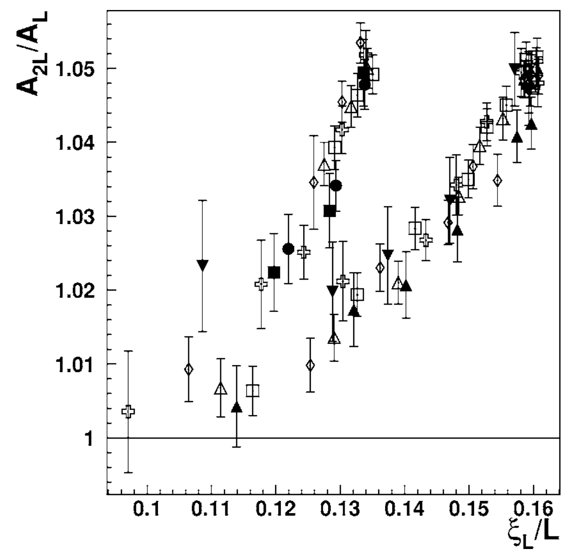

FIG. 3. FSS plot of the ratio ξ_L^2/χ_L in the two-dimensional RDLG with fixed $S_1=0.223$ (upper set of points) and 0.326 (lower set). Symbols are as in Fig. 2.

similar results, has been performed for the susceptibility $\chi_L \equiv G_\perp(q_1)$ [20]. We find more instructive to present data for the ratio $A_L \equiv \xi_L^2/\chi_L$, which is independent of ξ_L/L (for L large enough) whenever the critical exponent η vanishes.

In Fig. 3 we present the FSS data of this observable, for the two values of S_1 previously considered. In contrast to the DLG, where we got $A_{2L}/A_L \simeq 1$, here we see a pronounced and systematic dependence on ξ_L/L and on the actual value of S_1. For the two different values of $z^*(S_1)$ we do find the same value for $A_{2L}/A_L \simeq 1.05$, which is equal to 2^η [9] and leads to the estimate

$$\eta = 0.07(1). \quad (4)$$

Further evidence of differences in the critical behavior of the DLG and of the RDLG is provided by the transverse Binder

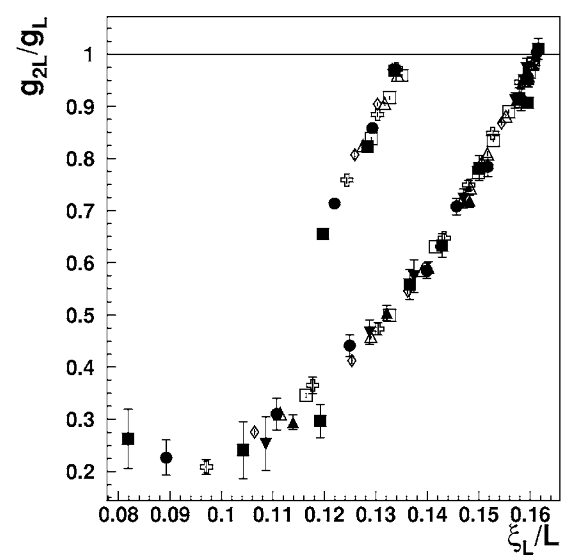

FIG. 4. FSS plot of the transverse Binder cumulant g_L in the two-dimensional RDLG with fixed $S_1=0.223$ (upper set of points) and $S_1=0.326$ (lower set). Symbols are as in Fig. 2.

cumulant $g_L \equiv -G_{\perp}^{(4)}(q_1,q_1,-q_1,-q_1)/[N_L G_{\perp}^2(q_1)]$, where $G_{\perp}^{(4)}$ is the Fourier transform of the four-point connected correlation function at a given β and lattice geometry, computed at the first allowed transverse momentum q_1, and N_L is the total number of spins in the lattice. The FSS plot in Fig. 4 shows clearly that for both the values of S_1 considered, $g_{2L}/g_L \simeq 1$ at the corresponding $z^*(S_1)$. Therefore, g_L at the critical point does not vanish in the thermodynamic limit, at variance with what has been found for the DLG [9]. The estimated values of the critical exponents [see Eqs. (3) and (4)] are in good agreement with previous numerical findings (although our result for η is smaller than that reported in Ref. [12]) and theoretical estimates based on the field-theoretical model of Ref. [11]. Moreover, the *universal* FSS functions for the correlation length agree with those computed in field theory at first order in an ϵ expansion around the upper critical dimension $d=3$ [20].

IV. CONCLUSIONS

We have shown that the FSS approach as devised in Ref. [14] is sensitive enough to distinguish clearly between the critical behavior of the DLG and of the RDLG, two systems which in particular geometries may exhibit quite similar behavior for relatively small volumes and not too close to the critical temperature. Therefore, on one side we have a sound numerical method to examine also nonequilibrium critical phenomena, on the other we have eventually established that the key features of these two models are different, as they do not belong to the same universality class. The agreement with the field-theoretical analysis of Refs. [3,11,12] suggests that indeed the leading critical behavior of the DLG is governed by the presence of a particle current whereas that one of the RDLG is dominated by strong anisotropy.

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